# The matrix Hamiltonian for hadrons and the role of negative-energy components

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**Abstract**

The world-line (Fock-Feynman-Schwinger) representation is used for quarks in arbitrary (vacuum and valence gluon) field to construct the relativistic Hamiltonian. After averaging the Green's function of the white $q\bar{q}$ system over gluon fields one obtains the relativistic Hamiltonian, which is matrix in spin indices and contains both positive and negative quark energies.

The role of the latter is studied in the example of the heavy-light meson and the standard einbein technic is extended to the case of the matrix Hamiltonian. Comparison with the Dirac equation shows a good agreement of the results. For arbitrary $q\bar{q}$ system the nondiagonal matrix Hamiltonian components are calculated through hyperfine interaction terms. A general discussion of the role of negative energy components is given in conclusion.

## 1 Introduction

The quest for the Hamiltonian which contains main features of QCD - confinement and Chiral Symmetry Breaking (CSB) exists from the very beginning, when fundamental field-theoretical QCD Hamiltonians, have been constructed in different gauges [1]. Unfortunately (nonlocal) confinement cannot be seen in these local FTh Hamiltonians and for practical purposes another sort of Hamiltonians – Effective Hamiltonians (EH) have been modelled containing minimal relativity and string-type potentials [2]. A lot of

information was obtained from these Hamiltonians and the general agreement of calculated meson and baryon masses with experiment is impressive [3], with some exceptions for mesons (e.g. pions, scalar nonets etc.) and for baryons (the Roper resonance and his companions, $\Lambda(1405)$ etc.).

The two main defects of effective Hamiltonians are that: i) The clear-cut derivation from the basic QCD Lagrangian was absent and therefore it is not clear what are approximations and how to improve EH systematically.

ii) Connected to that, the EH contains a large number of parameters in addition to the minimal QCD number: current quark masses and string tension (or $\Lambda_{QCD}$). Typically this additional number of parameters is more than ten for detailed spectrum calculation. The most important for hadron masses are constituent quark masses, $m_i$ and the overall negative constant $C_0$ of the order of several hundred MeV.

With this number of arbitrary parameters the QCD dynamics in hadrons cannot be fully understood and one needs another approach. This approach (it will be called the QCD string approach (SA)) was suggested more than a decade ago [4], where the SA Hamiltonian for spinless quarks was derived and Regge trajectories have been obtained both for mesons [4] baryons [5], and for glueballs [6]. Later on, the formalism was put on more rigorous basis in [7] and the einbein technic [8] was used in [7] to take into account the string moment of inertia and to obtain the correct Regge slope. In [4]-[7] the constituent mass was defined using the einbein technic through the string tension and current quark mass; the subsequent calculation of baryon magnetic moments [9] has confirmed the validity of this approach.

Another mysterious problem – of large negative constant $C_0$ – was understood recently in the framework of the same QCD string approach and $C_0$ was identified with the large quark self-energy term [10]. The latter is calculated through the string tension and quark current masses again without introduction of new parameters.

The Spin-Dependent (SD) part of the SA Hamiltonian was calculated earlier [11]. It was shown that even for light quarks one can calculate the leading SD terms without recurring to the $1/M$ expansion, but using instead the lowest (quadratic) field-correlator approximation [12] which works with accuracy of few percents [13].

The final form of the SA Hamiltonian was used to calculate the masses of light mesons [14, 15], heavy quarkonia [16, 17, 18], heavy-light mesons [19, 20, 21], baryons [22, 23, 24], glueballs [6, 25, 26], hybrids[27, 28, 29, 30], gluelumps[31]. For a review see also [32, 33, 34]. It is remarkable that in

most cases the agreement with known experimental data and lattice data is good, however only the minimal QCD set of parameters was used with addition of standard $\alpha_s(Q^2)$.

The most important exception in mesons from the agreement above was for pseudoscalars $(\pi, K, \eta, \eta')$ which need the chiral dynamics absent in the SA Hamiltonian. To overcome this discrepancy it was realized in [35, 36, 37] that the chiral dynamics brings a new tadpole term, which should be accounted for in computation of the Nambu-Goldstone spectrum. As a result the Gell-Mann-Oakes-Renner relation was found in [35, 36] and the quark condensate was computed [37] in terms of the SA Hamiltonian spectrum.

This connection allows to calculate the spectrum of Nambu-Goldstone mesons and their radial excitations in terms of the SA Hamiltonian spectrum, without introducing new parameters.

So far so good, but to proceed further one should look carefully into the approximations done and understand how to improve the SA Hamiltonian systematically.

The systematic procedure of the derivation of the SA Hamiltonian is given in [4]-[7] and discussed later in [38]-[40]. It contains three typical approximations: 1) Replacement of the Wilson loop by the minimal area expression and neglect of gluon excitation, which amounts to the neglect of mixing of a given hadron with all its hybrid excitations. As it was shown in [41] the effect of mixing is indeed small except for the cases of states almost degenerate in mass. 2) The use of the local Hamiltonian which appears in the limit of small gluon correlation length $\lambda$ (denoted as $T_g$ in most previous publications). Since $\lambda \cong 0.2$ fm and much smaller than typical hadron sizes, this limit is legitimate. 3) The use of only positive solution for the stationary point equations in the einbein variable, corresponding to the positive constituent quark mass. This latter approximation means neglect of the quark negative energy states, and it is the main point of the present investigation. As a result we shall obtain the Hamiltonian containing both positive and negative quark components, and estimate quantitatively the importance of the latter.

The paper is organized as follows. In the section 2 we derive the Green function for the $q\bar{q}$ system and consider in section 3 the one-body self-energy corrections for the quark and antiquark. Having fixed that, we turn in section 4 to the heavy-light $q\bar{q}$ interaction and derive the corresponding $q\bar{q}$ Hamiltonian in the full relativistic form, containing Negative Energy Components (NEC) and compare numerical results of matrix Hamiltonian with those for

the Dirac equation. In the section 5 the effects of NEC are derived for the general $q\bar{q}$ system. The last section is devoted to conclusions and outlook. Appendix 1 is devoted to the derivation of path integral form of the FFS type, in particular a novel first-order form of FFS is obtained for one particle in external nonabelian field using Weyl representation for $\gamma$ matrices. Appendix 2 contains details of self-energy correction.

## 2 The quark-antiquark Green function

We recapitulate here the steps done in derivation of the SA Hamiltonian [4, 7, 32].

One starts with the Fock-Feynman-Schwinger Representation (FFSR) for the quark (or valence gluon) Green's function in the Euclidean external gluonic fields [34, 41], which is exact and does not contain any approximation:

$$S(x,y) = (m+\hat{D})^{-1} = (m-\hat{D})\int_0^\infty ds (Dz)_{xy} e^{-K} P_A \exp(ig\int_y^x A_\mu dz_\mu) P_\sigma(x,y,s) \tag{1}$$

where $K$ is the kinetic energy term,

$$K = m^2 s + \frac{1}{4}\int_0^s d\tau \left(\frac{dz_\mu(\tau)}{d\tau}\right)^2 \tag{2}$$

and $m$ is the pole mass of quark, and $z_\mu(\tau)-$ the quark trajectory with end points $x$ and $y$ integrated over in $(Dz)_{xy}$.

The factor $P_\sigma(x,y,s)$ in (1) is generated by the quark spin (color-magnetic moment) and is equal

$$P_\sigma(x,y,s) = P_F \exp[g\int_0^s \sigma_{\mu\nu} F_{\mu\nu}(z(\tau))d\tau], \tag{3}$$

where $\sigma_{\mu\nu} = \frac{1}{4i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, and $P_F(P_A)$ in (3), (1) are respectively ordering operators of matrices $F_{\mu\nu}(A_\mu)$ along the path $z_\mu(\tau)$. In what follows the role of the operator $P_\sigma(x,y,s)$ will be crucial, and it is convenient to rewrite $\sigma_{\mu\nu}F_{\mu\nu}$ in $2\times 2$ notations

$$\sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{B} & \boldsymbol{\sigma}\mathbf{E} \\ \boldsymbol{\sigma}\mathbf{E} & \boldsymbol{\sigma}\mathbf{B} \end{pmatrix} \tag{4}$$

where $\boldsymbol{\sigma}$ are usual Pauli matrices.

The next step is the FFSR for the hadron Green's function, which for the case of the $q\bar{q}$ meson is

$$G_{q\bar{q}}(x,y;A) = \int_0^\infty ds \int_0^\infty ds' (Dz)_{xy} (Dz')_{xy} e^{-K-K'} tr(\Gamma(m-\hat{D}) W_{\sigma\sigma}(x,y) \bar{\Gamma}(m'-\hat{D}')) \tag{5}$$

where $\Gamma$ and $\bar{\Gamma} = \Gamma^+$ are $1, \gamma_\mu, \gamma_5, (\gamma_\mu\gamma_5), ...$, "tr" means trace operation both in Dirac and color indices, and

$$W_{\sigma\sigma'}(x,y) = P_A \exp(ig \int_{C(x,y)} A_\mu dz_\mu) P_\sigma(x,y,s) P'_\sigma(x,y,s'). \tag{6}$$

In (6) the closed contour $C(x,y)$ is along trajectories of quark $z_\mu(\tau)$ and antiquark $z'_\nu(\tau')$, and the ordering $P_A$ and $P_F$ in $P_\sigma, P'_\sigma$, is universal, i.e. $W_{\sigma\sigma'}(x,y)$ is the Wegner-Wilson loop with insertion of operators (4) along the contour $C(x,y)$ at the proper places.

The FFSR Eq. (5) is exact and is a functional of gluonic fields $A_\mu, F_{\mu\nu}$, which contain both perturbative and nonperturbative contributions, not specified at this level.

The next step, containing important approximation, is the averaging over gluonic fields, which yields the physical $q\bar{q}$ Green's function $G_{q\bar{q}}$

$$G_{q\bar{q}}(x,y) = \langle G_{q\bar{q}}(x,y;A)\rangle_A. \tag{7}$$

Here the averaging is done with the usual Euclidean weight $\exp(-action)$, containing all gauge-fixing, ghost terms, the exact form is inessential for what follows. To proceed it is convenient to use the nonabelian Stokes theorem [42] for the first factor on the r.h.s. in (6) and to rewrite the average of (6) as

$$\langle W_{\sigma\sigma'}(x,y)\rangle = \langle \exp ig \int d\pi_{\mu\nu}(z) F_{\mu\nu}(z)\rangle =$$

$$= \exp \sum_{n=1}^\infty \frac{(ig)^n}{n!} \int d\pi(1) ... \int d\pi(n) \langle\langle F(1)...F(n)\rangle\rangle \tag{8}$$

where

$$d\pi_{\mu\nu}(z) = ds_{\mu\nu}(z) - i\sigma_{\mu\nu} d\tau \tag{9}$$

and $ds_{\mu\nu}$ is the surface element. In (8) we have used the cluster expansion theorem and omitted indices of $d\pi(k), F(k)$ implying $d\pi(k) \equiv d\pi_{\mu_k\nu_k}(z) = ds_{\mu\nu}(u_k) - i\sigma_{\mu_k\nu_k} d\tau_k, F(k) \equiv F_{\mu_k\nu_k}(u_k, x_0) \equiv \Phi(x_0, u_k) F_{\mu_k\nu_k}(_k) \Phi(u_k, x_0)$, where $\Phi(x,y) = P \exp ig \int_y^x A_\mu dz_\mu$.

Eq. (8) is exact and therefore the r.h.s. does not depend on the choice of the surface, which is integrated over in $ds_{\mu\nu}(z)$. To proceed one makes at this point the approximation, keeping only lowest (quadratic) field correlator $\langle\langle F(i)F(k)\rangle\rangle$, while the surface is chosen to be the minimal area surface. As it was argued in [13], using comparison with lattice data, this approximation (sometimes called the Gaussian Approximation (GA)) has accuracy of few percent. The factors $(m-\hat{D})$ and $(m'-\hat{D}')$ in (5) need a special treatment in the process of averaging in (7), and as shown in Appendix 1 of [19] one can use a simple replacement,

$$m-\hat{D}\to m-i\hat{p},\quad p_\mu=\frac{1}{2}\left(\frac{dz_\mu}{d\tau}\right)_{\tau=s}. \qquad (10)$$

With the insertion of the cluster expansion (8) and the operator $(m-\hat{D})$ from Eq.(10) into the general expressions (7), (5), one fulfills the first step: the derivation of the physical $q\bar{q}$ Green's function in terms of vacuum correlators $\langle\langle F(1)...F(n)\rangle\rangle$. At this point it is important to discuss the separation of one-body (self-energy) and two-body terms in the interaction kernel (8), together with the separation of perturbative and nonperturbative contributions.

# 3 Quark self-energy in the confining background

It is clear that on physical grounds it is difficult to separate out the one-body (self-energy) contributions for the quark connected by the string to the antiquark. The situation here is different in the confining QCD from the nonconfining QED, since in the latter electron can be isolated and its selfenergy is a part of the renormalized electron mass operator. For the bound electron in an atom, the one body and two-body contributions to the interaction kernel can be separated in each order in $\alpha^n$, as it is done e.g. in the Bethe-Salpeter equation.

In case of QCD confinement cannot be excluded in any order of perturbative gluon exchanges and the separation seems to be impossible in principle. Nevertheless the world-line or FFS representation (1), (5), (7) suggests a possible way of separating out the one-body contributions, It is based on the distinguishing the perimeter $(L)$ law and area $(S_{min})$ law terms in the

Wegner-Wilson loop,

$$\langle W(C)\rangle = const\,\exp(-C_1 L - C_2 S_{min}), \tag{11}$$

where the one-body terms is associated with the coefficient $C_1$, while the two-body terms – with $C_2$. Going over from the Wegner-Wilson loop to the $q\bar{q}$ Green's function and the $q\bar{q}$ Hamiltonian, the situation however is becoming more complicated since $\langle G_{q\bar{q}}\rangle$ is an integral over all Wegner-Wilson loops, and typical loops ($q\bar{q}$ trajectories) have a finite average $q\bar{q}$ separation $\langle r\rangle$ and the same time length $T$, so that both perimeter and area-law terms contribute terms proportional to $T$. At this point the FFSR is helpful, since it allows to separate the Lorentz-invariant self-energy (SE) terms, $\Delta m^2$, which contribute to the Hamiltonian (see below and in [10]) as $\frac{\Delta m^2}{2\omega}$, where $2\omega = \frac{dt}{ds}$, and $t$ is the physical time, and $s$ , as in (1), is the proper time. One can see that the SE terms are multiplied by the effective length of trajectory, indeed $\Delta H dt = \frac{\Delta m^2}{2\omega} dt \sim \Delta m^2 ds$. At the same time the two-body terms in the Hamiltonian are proportional to $\omega$ (see below and in [10, 32]).

To calculate the SE terms explicitly we shall use the background perturbation theory [43, 44] with the separation of nonperturbative background field $B_\mu$ and valence (perturbative) gluon field $a_\mu$, so that the total vector potential $A_\mu$ can be written as

$$A_\mu = B_\mu + a_\mu. \tag{12}$$

The method [44] assumes the perturbative expansion in powers of $ga_\mu$, while $B_\mu$ enters via nonperturbative field correlators known from lattice [45] or analytic [31] calculations. Accordingly we separate the contributions to the quark SE (we prefer to use the $m^2$ instead of $m$, since $m^2$ appears in the Hamiltonian both in the FFS technic and after Foldy-Wouthuyzen diagonalization of the Dirac operators)

$$m^2(\mu) = m^2_{pert}(\mu) + \Delta m^2_{np}(\mu) + m^2_{int}. \tag{13}$$

Here $m^2_{pert}(\mu)$ is the pole mass and its connection to the $\overline{MS}$ mass is known to two loops (for a detailed discussion see the book [46])

$$m^{(pole)}_{pert}(\mu) = \overline{m}(\overline{m}^2)\left\{1 + \frac{C_F}{\pi}\alpha_s(m^{pole}) + O(\alpha_s^2)\right\}, \tag{14}$$

while the basic nonperturbative term $m^2_{np}$ was found in [10] (below we shall find a correction to this term) and the mixed perturbative-nonperturbative

contribution $m^2_{int}$ is yet to be calculated. We stress that $m^2(\mu)$ can be found in a gauge-invariant form only when it is computed inside the gauge-invariant $q\bar{q}$ or $3q$ Green's function, and in principle it may depend on the system where the quark is imbedded.

Since we are mostly interested in the case of light quarks, the perturbative mass evolution is small and unimportant, and the main term appears to be $\Delta m^2_{np}$, which we consider now.

Following [10] we consider the quadratic in $(\sigma F)$ term in (8) and expand the exponent to make explicit the resulting SE term (which is exponentiated after all, yielding additive contribution to the Hamiltonian). One has

$$\langle W_{\sigma\sigma'}(x,y)\rangle \cong \left\langle \left(1+\frac{g^2}{2}\sigma_{\mu\nu}\sigma_{\rho\lambda}\int_0^s d\tau\int_0^s d\tau' F_{\mu\nu}(z(\tau))F_{\rho\lambda}(z(\tau'))\right) W_0 + ...\right\rangle \tag{15}$$

where we have neglected the term proportional to $ds_{\mu\nu}ds_{\lambda\rho}$ since it contributes to the $q\bar{q}$ potential accounted for in Hamiltonian and not one-body SE terms; also the mixed terms ($\sim ds_{\mu\nu}\sigma_{\lambda\rho}$) contribute to the spin-orbit potentials, also taken into account in the Hamiltonian [11]. Here $W_0$ is the usual Wegner-Wilson loop without $(\sigma F)$ operators. The vacuum averaging in (15) yields in the Gaussian approximation (see Appendix of [11])

$$\langle F_{\mu\nu}F_{\rho\lambda}W_0\rangle = \{[\langle F_{\mu\nu}F_{\rho\lambda}\rangle - g^2\int ds_{\lambda\beta}\langle F_{\mu\nu}F_{\alpha\beta}\rangle\int ds_{\gamma\delta}\langle F_{\rho\lambda}F_{\gamma\delta}\rangle]\langle W_0\rangle\}. \tag{16}$$

Introducing scalar functions $D$ and $D_1$, as in [12] (we omit for simplicity parallel transporters $\Phi(u,v)$)

$$g^2\langle F_{\mu\nu}(n)F_{\rho\lambda}(v)\rangle = \hat{1}\{(\delta_{\mu\rho}\delta_{\nu\lambda}-\delta_{\mu\lambda}\delta_{\nu\rho})D(u-v)+$$

$$+\frac{1}{2}[\partial_\mu(h_\rho\delta_{\mu\lambda}-h_\lambda\delta_{\nu\rho})+\partial_\nu(h_\lambda\delta_{\mu\rho}-h_\rho\delta_{\mu\lambda})]D_1(u-v)\} \tag{17}$$

with $h_\mu = u_\mu - v_\mu$, one has

$$\sigma_{\mu\nu}\sigma_{\rho\lambda}\langle F_{\mu\nu}(z)F_{\rho\lambda}(z')W_0\rangle = 6[D(z-z')+D_1(z-z')]-$$

$$-4\int\sigma_{\alpha\beta}ds_{\alpha\beta}(u)D(u-z)\int\sigma_{\gamma\delta}ds_{\gamma\lambda}(v)D(v-z'), \tag{18}$$

where one should have in mind that in $ds_{\alpha\beta}$ it is always implied $\alpha<\beta$ both in (16) and consequently in (18).

Using now the identities

$$(Dz)_{xy} = (Dz)_{xu} d^4u (Dz)_{uv} d^4v (Dz)_{vy} \tag{19}$$

$$\int_0^\infty ds \int_0^s d\tau_1 \int_0^{\tau_1} d\tau_2 f(s,\tau_1,\tau_2) = \int_0^\infty ds \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 f(s+\tau_1+\tau_2, \tau_1+\tau_2, \tau_2) \tag{20}$$

one can rewrite (5), (7) with insertion of (15) as

$$G_{q\bar{q}}(x,y) = G^{(0)}_{q\bar{q}}(x,y) +$$
$$+ tr[\Gamma(m-\hat{D})\Delta_{xu}\sigma_{\mu\nu}d^4u\Delta_{uv}\sigma_{\rho\lambda}d^4v\Delta_{vy}\bar{\Gamma}(m'-\hat{D}')\Delta_{yx}\langle F_{\mu\nu}(u)F_{\rho\lambda}(v)W_0\rangle] \tag{21}$$

where we have defined

$$\Delta_{xu} \equiv \int_0^\infty ds e^{-K(s)} (Dz)_{xu}, \quad K(s) \equiv m^2 s + \frac{1}{4}\int_0^s \left(\frac{dz_\mu}{d\tau}\right)^2 d\tau \tag{22}$$

An alternative derivation is given in [10]. Note that $\langle W_0\rangle$ depends on trajectories entering in $\Delta_{zz'}$ in (21). One can now take into account that when $|x-u|$ is small, i.e. $|x-u| \lesssim T_g$ the influence of $\langle W_0\rangle$ on $\Delta_{xu}$ can be neglected, since $\langle W_0\rangle$ is a smooth function of its boundaries, varying when they are deformed at the scale larger than $T_g$, while $\Delta_{xu}$ is singular for small $|x-u|$. Indeed in this limit neglecting the presence of $\langle W_0\rangle$ one obtains

$$\Delta^{(0)}_{xu} = \frac{m}{4\pi^2}\frac{K_1(m|x-u|)}{|x-u|}. \tag{23}$$

For large $|x-u|$ one can use the fact, that the product of the spinless quark Green's function $\Delta_{xy}$ and that of the spinless antiquark $\Delta_{yx}$ together with $\langle W_0\rangle$ yield the asymptotics of the total meson mass $M_0$ without spin contributions, and without self-energy corrections

$$\int \Delta_{xy}\Delta_{yx}\langle W_0\rangle \sim \exp(-M_0|x-y|). \tag{24}$$

Therefore we shall use for $\Delta_{xy}$ at large $|x-y|$ the interpolation form

$$\Delta_{xu}(x) \cong \frac{\bar{m}K_1(\bar{m}|x|)}{4\pi^2|x|}, \quad \bar{m} = m + \tilde{M}_0, \quad \tilde{M}_0 \approx \frac{M_0}{2}. \tag{25}$$

(We do not need a high accuracy of (25) since it will enter in the small correction term).

Now inserting (18) into (21) one obtains the following combination.

$$J(u,v) = 6[D(u-v) + D_1(u-v)]\Delta_{uv} -$$

$$- 4\sigma_{\alpha\beta}\sigma_{\gamma\delta} \int ds_{\alpha\beta}(z) D(z-u) \int ds_{\gamma\delta}(w) D(w-v). \qquad (26)$$

In the first term on the r.h.s. of (26) $\Delta_{uv}$ enters with the factors $D(u-v)$, $D_1(u-v)$ which fall off with small correlation length $T_g$. Therefore in [10] $\Delta_{uv}$ was taken as $\Delta^{(0)}_{uv}$. Here we tend to improve this result by taking into account the asymptotic fall-off of $\Delta_{uv}$ as in (24), (25). This can be done replacing in the free propagator by the $\Delta_{wv}$ from (25) so that the asymptotics both at small and large distances is reproduced.

Identifying $m^2_{np}$ from the expansion

$$(m^2+\Delta m^2_{np}-D^2)^{-1} = (m^2-D^2)^{-1}-(m^2-D^2)^{-1}\Delta m^2_{np}(m^2-D^2)^{-1}+... \qquad (27)$$

one obtains

$$\Delta m^2_{np} = -\int d^4w \frac{\bar{m}K_1(\bar{m}|w|)}{4\pi^2|w|} 6(D(|w|)+D_1(|w|))+\sigma^2 \int \Delta_{xu} d^4(x-u),. \qquad (28)$$

We take the lattice estimate (for the quenched case [45]), $D_1 \cong \frac{1}{3}D$ and the relation [12] $\sigma = \frac{1}{2}\int D(z)d^2z$, and obtain

$$\Delta m^2_{np} = -\frac{4\sigma}{\pi}\varphi(t) + \frac{\sigma^2}{2(m+\tilde{M}_0)^2}; \quad t = (m+\tilde{M}_0)T_g. \qquad (29)$$

Here $\varphi(t)$ is given in Appendix 2; it is normalized as $\varphi(0) = 1$. The first term on the r.h.s. of (29) coincides with the result [10] when $\tilde{M}_0$ is neglected (note however that coefficients before $D$ and $D_1$ in [10] have a misprint, and should be replaced by those in (26)), while the last term in (29), which is a correction to the first, is new. To understand the role of this term in creating the total mass of the meson (still without Coulomb correction and spin-dependent terms), we take the case of the heavy-light meson, i.e. when the quark is moving in the field of an infinitely heavy antiquark. The Hamiltonian in this case is written as in [19]-[21] with the SE term treated as in [10], namely

$$H_0(\omega) = \frac{m^2+\Delta m^2_{np}}{2\omega} + M_0(\omega) \qquad (30)$$

where $\tilde{M}_0(\omega) = \frac{\omega}{2} + \varepsilon(\omega), \varepsilon(\omega) = \frac{\sigma^{2/3} a(n)}{(2\omega)^{1/3}}; a(0) = 2.338$. Taking into account (29) the resulting expression for $H_0(\omega)$ is

$$H_0(\omega) = \left( -\frac{4\sigma\varphi(t)}{\pi} + \frac{\sigma^2}{2(\tilde{M}_0 + m)^2} \right) \frac{1}{2\omega} + \tilde{M}_0(\omega). \tag{31}$$

Now $\omega$ should be found from the equation [4, 7, 32]

$$\frac{\partial H_0(\omega)}{\partial \omega}|_{\omega=\omega_0} = 0 \tag{32}$$

Neglecting in (32) the contribution of the SE term, one obtains the values of $\omega_0$ and $\tilde{M}_0, H_0(\omega_0) \equiv M$ as in [4] which are shown in Table 1 all masses are given in GeV.

**Table 1 Mass eigenvalues (in GeV) according to Eqs. (31), (32) for different values of $\alpha_s$**

| $\alpha_s$ | $\omega_0$ | $\varepsilon$ | $\tilde{M}_0$ | $\varphi(t)$ | $\Delta M_{SE}$ | $M = \tilde{M}_0 + \Delta M_{SE}$ |
|---|---|---|---|---|---|---|
| 0 | 0.448 | 0.735 | 0.96 | 0.416 | -0.106 | 0.854 |
| 0.3 | 0.546 | 0.498 | 0.771 | 0.5 | -0.105 | 0.666 |
| 0.39 | 0.594 | 0.407 | 0.704 | 0.525 | -0.101 | 0.603 |

For $\alpha_s > 0$ the values $\varepsilon(\omega)$, $\tilde{M}_0 = \frac{\omega}{2} + \varepsilon(\omega)$ have been calculated in [4] taking the color Coulomb term $-\frac{4}{3}\frac{\alpha_s}{r}$ into account, while $\Delta M_{SE} = -\frac{2\sigma}{\pi\omega_0}\varphi(t)$ and $t = T_g \tilde{M}_0$, $T_g = 1$ GeV$^{-1}$. One can see that $\Delta M_{SE}$ is rather stable and gives a correction around 15% to the total mass. The correction of the second term on the r.h.s. of (29) to the total $\Delta m^2$ is of the order of 7% for $m = 0$ , so that the earlier calculations made without this term in [14]-[18] and [22]-[24] would be modified by few percent.

As the next comparison one can take the solution of Dirac equation for the heavy-light meson with confining and color Coulomb term present. In this case the SE term is absent in the first order Dirac Hamiltonian (in contrast to the second-order SA Hamiltonian, obtained from FFSR). The results of calculations, performed in [47, 48] are shown in Table 2. In this case to compare with the SA Hamiltonian (31) one should take $T_g$ in $\varphi(t)$ equal to zero, since the linear confining potential is obtained in this limit

(while for $T_g \neq 0$ there appear corrections to the linear potential calculated in [48]). Hence in (31) one puts $\varphi(t=0)=1$ and neglects as before the second correction term in brackets on the r.h.s. As a result one obtains for $\sigma$=0.16 GeV$^2$ (all masses are given in GeV).

**Table 2 Masses and self-energy corrections according to Eq. (31) in comparison with eigenvalues of Dirac equation.**

| $\alpha_s$ | 0 | 0.3 | 0.39 |
|---|---|---|---|
| $\Delta M_{SE}$ | -0.227 | -0.186 | -0.171 |
| $M=\tilde{M}_0+\Delta M_{SE}$ | 0.733 | 0.585 | 0.533 |
| $M_D$ | 0.65 | 0.465 | 0.401 |

In the last line the Dirac eigenvalues from [47, 48] are given to be compared with the eigenvalues of (31) in the next line. One can see that Dirac eigenvalues are about 100 MeV lower. This difference can be attributed to the fact that in Dirac equation both positive and negative eigenvalues (the latter corresponding to the backward-in-time motion of quark) are taken into account, while in (31) only positive values of $\omega_0$ are considered. In the next section we shall discuss how the negative modes (negative solutions for $\omega_0$) can be included in the SA Hamiltonian.

# 4 The matrix Hamiltonian for the heavy-light $q\bar{q}$ system

We start with the Hamiltonian for the free Dirac particle $\hat{H}=m\beta+\boldsymbol{\alpha}\mathbf{p}$, which can be diagonalized using Foldy-Wouthuyzen (FW) procedure

$$\hat{H}=U^+\hat{H}_dU,\quad \hat{H}_d=\left(\begin{array}{ll}\sqrt{\mathbf{p}^2+m^2} & 0\\ 0 & -\sqrt{\mathbf{p}^2+m^2}\end{array}\right). \tag{33}$$

As it is explained in detail in Appendix 1 the free Green's function can be written in terms of $\hat{H}_d$ as

$$S(t)=i\beta U\left(\begin{array}{ll}\theta(t) & 0\\ 0 & -\theta(-t)\end{array}\right)e^{-i\hat{H}_dt}U^+. \tag{34}$$

In the Appendix 1 also the case of the Weyl representation is discussed for the Dirac particle in the external fields, which gives a representation similar to (34), i.e. having the diagonal Hamiltonian of the form of $\hat{H}_d$ in the exponent for the time-dependent Green's function.

We now turn to the FFS form of the quark Green's function in the external gluonic field, written with the help of the einbein function $\omega$ [4, 7] (this function was previously denoted as $\mu$ in most papers)

$$S_q(x,y) = \int D\omega (D^3 z)_{\mathbf{xy}} e^{-\int_0^T \left(\frac{m^2}{2\omega}+\frac{\omega}{2}+\frac{\omega \dot{z}_i^2}{2}\right)dt + ig\int_y^x A_\mu dz_\mu + g\int_0^T \sigma F \frac{dt}{2\omega}}. \tag{35}$$

After vacuum averaging this function can be associated with the Green function of the heavy-light meson.

Here $D\omega$ is the path integration over functions $\omega(t)$, which in our formalism [4, 7] is calculated by the stationary point (steepest descent) method, after going over to the Hamiltonian form instead of the Lagrangian path integral form of (35), namely,

$$S_q(x,y) = \left\langle x \left| \int D\omega e^{-i\int_0^{T_M} \left(H_0(\omega) - g\frac{\sigma F}{2\omega}\right) dt_M} \right| y \right\rangle, \tag{36}$$

where we have changed from the Euclidean time $t$ to the Minkowskian time $t_M = -it$, and

$$H_0(\omega) = \frac{m^2}{2\omega} + \frac{\omega}{2} + \frac{\mathbf{p}^2}{2\omega} + \sigma r. \tag{37}$$

Solving the Schroedinger-type equation

$$\left(\frac{\mathbf{p}^2}{2\omega} + \sigma r\right)\varphi_n = \varepsilon_n(\omega)\varphi_n, \tag{38}$$

one obtains

$$\varepsilon_n(\omega) = \frac{\sigma^{2/3}}{(2\omega)^{1/3}} a_n \tag{39}$$

where $a_n, n = 0, 1, 2, ...$ is the set of zero of Eiry functions, $a_0 \cong 2.338$. As a result for $L = 0$ one has the eigenvalues $E_n^{(0)}(\omega)$ of $H_0(\omega)$, equal to

$$E_n^{(0)}(\omega) = \frac{m^2}{2\omega} + \frac{\omega}{2} + \frac{\sigma^{2/3}}{(2\omega)^{1/3}} a_n. \tag{40}$$

In our previous calculations of the spectrum the stationary point in the integration over $D_\omega$ was taken at the positive solution of the equation

$$\frac{\partial E_n^{(0)}(\omega)}{\partial\omega}\Big|_{\omega=\omega_n^{(0)}}=0,\quad \omega_n^{(0)}>0. \tag{41}$$

However there is a negative solution, at $\omega_n=-\omega_n^{(0)}$, which was neglected in all previous calculations.

The Hamiltonian (37) refers actually to the gauge-invariant system of a heavy-light $q\bar{Q}$ system, where the infinitely heavy quark $\bar{Q}$ propagates along the time axis and is situated at the spacial origin.

In line with the Hamiltonian (33), we can write the total Hamiltonian of the $q\bar{Q}$ system with the lower Dirac components as

$$\hat{H}_{q\bar{Q}}=\begin{pmatrix} h_0 & h_{+-}\\ h_{-+} & -h_0\end{pmatrix}, h_0\varphi=E_n^{(0)}\varphi \tag{42}$$

In $h_0$ the spin-dependent term $\frac{g(\sigma F)}{2\omega}$ in the exponent of (36) does not contribute to the diagonal part of (42) for $s$-wave states of heavy-light mesons, except for the diagonal SE term considered in the previous section (this term can be added replacing $m^2$ in (37), (40) by $m^2+\Delta m^2_{np}$).

Now we turn to the calculation of the terms $h_{+-}=h^*_{-+}$. To this end one can use Eqs.(5), (6), (8) having in mind that for the heavy-light meson the Green's function of the heavy quark reduces to the parallel transporter $\Phi(x,y)$ along the straight line and the factors $ds'(Dz')_{xy}e^{-K'}$ and absent. Having in mind (4) one must calculate the factor

$$\langle W^{(2)}_{\sigma\sigma'}(x,y)\rangle=\exp\left[-\frac{g^2}{2}\int d\pi(1)d\pi(2)\langle F(1)F(2)\rangle\right]. \tag{43}$$

In the product $d\pi(1)d\pi(2)$ the term $ds(1)ds(2)$ contributes to the linear interaction and is present in $H_0(\omega)$, the term $d\tau(1)d\tau(2)$ was calculated in the previous section and in [10] and was taken into account in $\Delta m^2_{np}$. The mixed terms can be written as

$$\exp\left(-g^2\int ds_{\mu\nu}(u)d\tau\left\langle F_{\mu\nu}(u)\begin{pmatrix}\boldsymbol{\sigma}\mathbf{B} & \boldsymbol{\sigma}\mathbf{E}\\ \boldsymbol{\sigma}\mathbf{E} & \boldsymbol{\sigma}\mathbf{B}\end{pmatrix}_{z(\tau)}\right\rangle\right). \tag{44}$$

The diagonal terms in (44) contribute to the spin-orbit interaction, computed in [11], and vanish for the $s$-states, while the nondiagonal terms contribute

to $h_{+-}, h_{-+}$ and calculated below. Writing

$$ds_{\mu\nu}F_{\mu\nu}(u) = ds_{i4}E_i(u) + ds_{ik}F_{ik} = n_i d^2 u E_i(u) + d\mathbf{s}\mathbf{B}, \quad \mathbf{n} = \frac{\mathbf{u}}{|\mathbf{u}|}, \tag{45}$$

one should average this term with nondiagonal component $\boldsymbol{\sigma}\mathbf{E}$ and neglect the last term, since the correlator (45) $\langle B_i E_k \rangle$ is proportional to $\frac{\partial D_1}{\partial u_l}$ and small, as a result one obtains the integral

$$\int d^2 u \langle (\mathbf{n}\mathbf{E}(u))(\boldsymbol{\sigma}\mathbf{E}(z(\tau)) \rangle = \boldsymbol{\sigma}\mathbf{n} \int d^2 u D(u-z) = \boldsymbol{\sigma}\mathbf{n}\sigma \tag{46}$$

where we have taken into account the definition $\sigma = \frac{1}{2}\int d^2 u D(u)$, and the fact that $z(\tau)$ lies on the quark trajectory, which is the boundary of the integration surface.

Finally one must replace $d\tau$ in (44) by $dt$, having in mind that upper matrix elements refer to the positive time evolution, while lower ones (corresponding to the negative energy eigenvalues) refer to the negative (backward) in time evolution, viz.

$$\int d\tau \begin{pmatrix} a & c \\ d & b \end{pmatrix} = \begin{pmatrix} \int \frac{dt}{2\omega} a & \int \frac{dt}{2\omega} c \\ -\int \frac{dt}{2\omega} d & -\int \frac{dt}{2\omega} b \end{pmatrix}. \tag{47}$$

As a consequence from (44) and (46) one obtains

$$h_{+-} = \frac{i\sigma}{2\omega}(\boldsymbol{\sigma}\mathbf{n}), \quad h_{-+} = -\frac{i\sigma}{2\omega}(\boldsymbol{\sigma}\mathbf{n}). \tag{48}$$

The energy eigenvalues of the matrix Hamiltonian (42) are obtained in the usual way from the equation

$$\det \begin{pmatrix} h_0 - E, & h_{+-} \\ h_{-+}, & h_0 - E \end{pmatrix} = 0, \quad h_0 \equiv E_n^{(0)}(\omega) \tag{49}$$

which yields

$$E = \pm\sqrt{h_0^2 + \left(\frac{\sigma}{2\omega}\right)^2} \tag{50}$$

where $\omega$ should be found from the condition $\frac{\partial E}{\partial \omega}|_{\omega=\omega_0} = 0$, which replaces the old condition (41), and can be rewritten as

$$2h_0 h_0' - \frac{\sigma^2}{2\omega_0^3} = 0. \tag{51}$$

Writing $h_0 = \frac{m^2-\Delta}{2\omega} + M_0(\omega)$ , with $\Delta = \frac{4\sigma}{\pi}\varphi(t)$, one can rewrite (51) for the case $m = 0, \quad T_g = 0, \quad \varphi \equiv 1$,

$$\omega_0 = \bar{\omega}_0 \left\{ 1 - 2\left( \frac{c_1}{\omega_0^2} - \Delta \right) \left( \frac{\omega_0}{\bar{\omega}_0} \right)^{4/3} \right\}^{3/4} \tag{52}$$

where $c_1 = \frac{2\sigma}{\pi}, \quad \bar{\omega}_0 = \sqrt{2\sigma}\left(\frac{a}{3}\right)^{3/4}, \Delta = \frac{\sigma^2}{4\omega_0^3 h_0(\omega_0)}$.

Solving (52) one obtains for $\sigma = 0.18$ GeV$^2$

$$h_0(\omega_0) \approx 0.56 \text{ GeV}, \quad \omega_0 \approx 0.21 \text{ GeV} \tag{53}$$

and the energy eigenvalue (50) equal to

$$E_0 = E(\omega_0) = \pm 0.70 \text{ GeV} \tag{54}$$

which should be compared with the Dirac eigenvalue from Table 2, $E_D = 1.619\sqrt{\sigma} = 0.686$ GeV. Thus taking into account matrix structure of the Hamiltonian diminishes eigenvalue by $\sim 0.1$ GeV and yields values in good agreement with independent calculation of Dirac equation.

It is of interest to compare this Hamiltonian with the Hamiltonian of the Dirac equation for a light quark in the static source of linear potential, considered in [47, 48].

$$\hat{H}_{Dirac} = \boldsymbol{\alpha}\mathbf{p} + \beta(m + \sigma r). \tag{55}$$

For the solution $\psi_n(\mathbf{r})$ represented in the form [47]

$$\psi_n(\bar{r}) = \frac{1}{r} \begin{pmatrix} G_n(r)\Omega_{jlM} \\ iF_n(r)\Omega_{jl'M} \end{pmatrix} \tag{56}$$

the equation $H_{Dirac}\psi_n = \varepsilon_n\psi_n$ assumes the form

$$\left\{ \begin{array}{l} \frac{dG_n}{dr} + \frac{\kappa}{r}G_n - (\varepsilon_n + m + \sigma r)F_n = 0 \\ \frac{dF_n}{dr} + \frac{\kappa}{r}F_n - (\varepsilon_n - m - \sigma r)G_n = 0 \end{array} \right. \tag{57}$$

where $\kappa(j,l) = (j + \frac{1}{2})sgn(l-j)$.

Equations (57) are invariant under the substitution $(\varepsilon_n, G_n, F_n, \kappa) \leftrightarrow (-\varepsilon_n, F_n, G_n, -\kappa)$. This means that for every solution with $\varepsilon_n > 0$, and $\kappa$ having the form (56) there exists another solution of the form

$$\psi_{-\varepsilon_n}(r) = \frac{1}{r} \begin{pmatrix} F_n(r)\Omega_{jl'M} \\ iG_n(r)\Omega_{jlM} \end{pmatrix} \tag{58}$$

which has eigenvalue $-\varepsilon_n, -\kappa$.

Following the idea of the FW transformation leading to (33) one can also assume that the Hamiltonian (55) can be diagonalized to the form

$$\hat{H}_{Dirac} \to U^+ \begin{pmatrix} \hat{h}(\kappa) & \\ & -\hat{h}(-\kappa) \end{pmatrix} U,$$

where $\hat{h}(\kappa)\varphi_n^\kappa = \varepsilon_n \varphi_n^\kappa$ and $\hat{h}(-\kappa)\varphi_n^{-\kappa} = \varepsilon_n \varphi_n^{-\kappa}$.

This brings us to the eigenvalue matrix (42). One special feature of this representation is that the states $\psi_{\varepsilon_n}$ and $\psi_{-\varepsilon_n}$ have different parities.

# 5 Negative energy states for the $q\bar{q}$ mesons

We are now considering the $q\bar{q}$ meson states made of light quarks. The Hamiltonian for positive energy states was used repeatedly (see [32] for details) and has the form

$$H_0(\omega_1, \omega_2) = \frac{m_1^2 - \Delta_1}{2\omega_1} + \frac{m_2^2 - \Delta_2}{2\omega_2} + \frac{\omega_1 + \omega_2}{2} + \frac{\mathbf{p}^2}{2\tilde{\omega}} + \sigma r \qquad (59)$$

where $\tilde{\omega} = \frac{\omega_1 \omega_2}{\omega_1 + \omega_2}$, $\Delta_i = \frac{4\sigma}{\pi}\varphi(t_i)$, $t_i = (m_i + \tilde{M}_0)T_g$. Turning now to the spin-dependent term $(\sigma F)$ in (35) and (8), one remarks that in addition to the one-quark corrections considered in the previous section one has also the spin-spin term, previously treated in [11] and yielding the hyperfine interaction, namely the term $V_4$. However in the derivation only the diagonal components of the matrix $\langle \sigma^{(1)}_{\mu\nu} F_{\mu\nu} \sigma^{(2)}_{\alpha\beta} F_{\alpha\beta} \rangle$ have been taken into account and now we shall look carefully into the nondiagonal terms.

From (8) one has the following contribution (Note that for antiquark the spin operator $\sigma^{(2)}_{\mu\nu}$ enters with the spin opposite to $\sigma^{(1)}_{\mu\nu}$)

$$\exp\left\{ -\frac{1}{2} \int_0^{s_1} d\tau_1 \int_0^{s_2} d\tau_2 g^2 \left\langle \begin{pmatrix} \boldsymbol{\sigma}^{(1)}\mathbf{B} & \boldsymbol{\sigma}^{(1)}\mathbf{E} \\ \boldsymbol{\sigma}^{(1)}\mathbf{E} & \boldsymbol{\sigma}^{(1)}\mathbf{B} \end{pmatrix}_{z(\tau_1)} \begin{pmatrix} \boldsymbol{\sigma}^{(2)}\mathbf{B} & \boldsymbol{\sigma}^{(2)}\mathbf{E} \\ \boldsymbol{\sigma}^{(2)}\mathbf{E} & \boldsymbol{\sigma}^{(2)}\mathbf{B} \end{pmatrix}_{z(\tau_2)} \right\rangle \right\} \qquad (60)$$

and one should replace as usual $d\tau_i = \pm \frac{dt_i}{2\omega_i}$.

As a result on obtains the following spin-spin terms in the Hamiltonian: i) from the product of diagonal components $\langle \boldsymbol{\sigma}^{(1)}\mathbf{B}\boldsymbol{\sigma}^{(2)}\mathbf{B} \rangle$ one has the usual hyperfine interaction [11, 32]

$$\hat{V}^{(diag)}_{hf}(r) = \frac{\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}}{12\omega_1\omega_2} \int_{-\infty}^{\infty} d\nu \left[ 3D(r,\nu) + 3D_1(r,\nu) + 2\mathbf{r}^2 \frac{\partial D_1}{\partial r^2} \right] \qquad (61)$$

where $D(r,\nu)=D(\sqrt{r^2+\nu^2})$, and $\mathbf{r}=\mathbf{z}_1(t)-\mathbf{z}_2(t)$ is the quark-antiquark distance, and we have used (17) to calculate $\langle B_i(u)B_k(v)\rangle$. Eq. (61) contains both perturbative and nonperturbative contributions, the latter have been calculated in [11] and found to be much smaller than the perturbative ones, which can be easily calculated using the lowest order form of $D_1$ [11]

$$D_1^{pert}(x)=\frac{16}{3\pi}\frac{\alpha_s}{x^4}+O(\alpha_s^2) \tag{62}$$

which gives the standard result

$$\hat{V}_{nf}^{(diag)}=\frac{8\pi\alpha_s\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\delta^{(3)}(\mathbf{r})}{9\omega_1\omega_2}. \tag{63}$$

The matrix element of (63) can be written as

$$\langle\hat{V}_{nf}^{(diag)}\rangle=\frac{2\alpha_s\langle\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\rangle}{9\omega_1\omega_2}R_n^2(0)=\frac{4\alpha_s\tilde{\sigma}}{9(\omega_1+\omega_2)}\left(\begin{array}{ll}-3, & S=0\\ +1, & S=1\end{array}\right) \tag{64}$$

where $\tilde{\sigma}\equiv\sigma+\frac{4}{3}\alpha_s\langle r^{-2}\rangle$, for more details see Appendix 3 of [37].

ii) We now turn to the product of nondiagonal terms in (60), which can be written similarly to (61) as

$$\hat{V}_{hf}^{(nond)}(r)=\frac{\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}}{12\omega_1\omega_2}\int_{-\infty}^{\infty}d\nu\left[3D+3D_1+(3\nu^2+r^2)\frac{\partial D_1}{\partial r^2}\right]. \tag{65}$$

Keeping again only perturbative contribution, one easily obtains

$$\hat{V}_{hf}^{(nond)}=-\hat{V}_{hf}^{(diag)}. \tag{66}$$

Consider now the total Hamiltonian

$$\hat{H}=(H_0(\omega_1,\omega_2)+\hat{V}_{hf}^{(diag)})\hat{1}_1\hat{1}_2+\hat{V}_{hf}^{(nond)}(\gamma_5)_1(\gamma_5)_2. \tag{67}$$

The energy eigenvalues can be found in the same way, as it was done in the previous section

$$E^{(\omega_1,\omega_2)}=\pm\sqrt{\left(H_0(\omega_1,\omega_2)+\hat{V}_{hf}^{(diag)}\right)^2+\left(\hat{V}_{hf}^{(nond)}\right)^2}. \tag{68}$$

To illustrate the general result (68) we take the case of massless quarks, and write the energy as

$$E(\omega)=\pm\sqrt{\left(h_0(\omega)+\frac{c_\sigma}{\omega}\right)^2+\frac{c_\sigma^2}{\omega^2}} \tag{69}$$

where $h_0(\omega)$ is the eigenvalue of $H_0(\omega_1, \omega_2), \omega_1 = \omega_2$

$$h_0(\omega) = -\frac{\delta}{\omega} + \omega + \frac{c}{\omega^{1/3}}; c = \sigma^{2/3} a(n), a(0) = 2.338. \tag{70}$$

Also we have defined

$$c_\sigma = \frac{2\alpha_s \tilde{\sigma}}{9} \begin{pmatrix} -3, & S = 0 \\ 1, & S = 1 \end{pmatrix}. \tag{71}$$

The crucial point is now that $\omega$ is to be found as before from the minimum of $E(\omega)$ (we assume that minimization of the eigenvalue $E(\omega)$ instead of the operator Hamiltonian $H(\omega_1, \omega_2)$ brings about a small correction as it was in the case of the one-channel Hamiltonian, see [7] and numerical analysis in the second ref. of [7]). This section served as an illustration of positive-negative state mixing due to hyperfine interaction in $\bar{q}q$ mesons. For the lack of space the detailed analysis of the corresponding change in the spectrum will be published elsewhere.

# 6 Conclusions

In this paper the systematic discussion is started of the role of negative energy components (NEC) for quark bound states. The NEC are automatically taken into account in the one-body Dirac or Bethe-Salpeter formalism. In the latter case however the Bethe-Salpeter wave-function contains for the $q\bar{q}$ meson at least eight independent components and their relative role can be studied only numerically. A didicated analysis of NEC was done in the quasipotential approximation of the Bethe-Salpeter equations [49] and effects of NEC was found to be significant for the spectrum of mesons. Recently another approach was introduced in [50, 48] and developed further in [51, 52], called the Method of Dirac Orbitals (MDO), where the quark bound state is expanded in a series of products of individual one-body Dirac states. In this case also the NEC effects are taken into account, but other approximations are usually done (c.m. motion, higher components) which require a cross-check of all results and comparison to other formalisms.

The Hamiltonian formalism and in particular the SA Hamiltonian is physically transparent and mathematically simple, it reduces to the popular Relativistic Quark Model (RQM) Hamiltonian in its simplest form (when string

motion is neglected) and therefore it is necessary to understand the role of NEC in the Hamiltonian form.

This is done in the present paper using the simplest bound system – the heavy-light meson, where the heavy quark plays the role of external field and therefore results can be compared to those of Dirac equation. The comparison proceeds in two steps. Firstly one takes in the Hamiltonian the one-body self-energy terms which do not mix positive energy components and NEC. Here a new correction term was obtained in section 3, Eq. (41) in addition to the old one [10] which gives around 7% of the total. Secondly, the NEC mixing appears due to the nondiagonal Hamiltonian matrix elements. The stationary point condition for the einbein variable $\omega$ should now be applied to the eigenvalues of the total matrix Hamiltonian $E(\omega)$

$$\hat{H} = \left( \begin{array}{cc} h_0 & h_{+-} \\ h_{-+} & -h_0 \end{array} \right), \quad det(\hat{H} - E(\omega)) = 0, \quad \frac{\partial E(\omega)}{\partial \omega} = 0. \tag{72}$$

The resulting stationary values $\omega_0$ and $E(\omega_0)$ are in good agreement with the eigenvalue of the Dirac operator $E_D$.

$$E(\omega_0) \cong E_D. \tag{73}$$

This procedure justifies the stationary analysis with respect to $\omega$, since for $h_0(\omega)$ the stationary point does not exist for light quarks if the self-energy term $-\frac{\Delta m^2}{2\omega}$ is included in $h_0(\omega)$, while for $E(\omega)$ the stationary point $\omega \equiv \omega_0$ always exists. In chapter 5 the first step is done for arbitrary $q\bar{q}$ systems, and the matrix Hamiltonian is written down explicitly. The main lesson here is that NEC are mixed up by the hyperfine interaction and the doubly nondiagonal (for both quark and antiquark) terms can be calculated explicitly. The further analysis and numerous applications are relegated for the lack of space to future publications. However already at this stage it is clear that NEC are very important for the structure of the wave-functions and eigenvalues of mesons. This is probably even more so for baryons, where NEC are responsible for the correct relativistic structure of baryon wave-functions, which is clearly seen in the values of the $g_A/g_V$ ratio [53].

The author is grateful to A.B.Kaidalov, I.M.Narodetsky, J.A.Tjon, M.A.Trusov and J.Weda and members of ITEP seminar theory group for useful discussions.

The work is supported by the Federal Program of the Russian Ministry of industry, Science and Technology No.40.052.1.1.1112.

## Appendix 1

In this appendix we shall derive several representations for the quark Green's function in the external nonabelian field. We start with the standard Fock-Feynman-Schwinger Representation (FFSR) as a warm up. To this end one writes first the proper-time representation in the Euclidean space-time

$$S = (m + \hat{D})^{-1} = (m - \hat{D})(m^2 - \hat{D}^2)^{-1} = (m - \hat{D}) \int_0^\infty ds e^{-s(m^2 - \hat{D}^2)}. \quad \text{(A.1)}$$

Now one transforms (A.1) to the path-integral as follows

$$\langle x| \int_0^\infty ds e^{sD_\mu^2} |y\rangle = \langle x| e^{\varepsilon D_\mu^2(N)} |x_{n-1}\rangle \langle x_{n-1}| e^{\varepsilon(D_\mu^2(N-1))} |x_{n-2}\rangle ... \langle x_1| e^{\varepsilon D_\mu^2(1)} |y\rangle. \quad \text{(A.2)}$$

In (A.2) the integration over all $d^4x_1...d^4x_{N-1}$ is implied and the relation $s = \varepsilon N$ is used. Consider now one piece of the path in (A.2) and write

$$I_{n,n-1} \equiv \langle x_n| e^{\varepsilon(\partial_\mu - igA_\mu)^2} |x_{n-1}\rangle = \langle x_n|p\rangle \frac{d^4p}{(2\pi)^4} e^{\varepsilon(\partial_\mu - igA_\mu(\frac{x_n+x_{n-1}}{2}))^2} \langle p|x_{n-1}\rangle =$$

$$= \frac{d^4p}{(2\pi^4} e^{ip(x_n - x_{n-1}) - \varepsilon(p_\mu - gA_\mu(\frac{x_n+x_{n-1}}{2}))^2}. \quad \text{(A.3)}$$

Integration over $d^4p$ in (A.3) gives

$$I_{n,n-1} = \frac{1}{(4\pi\varepsilon)^2} e^{-\frac{(\Delta x)^2}{4\varepsilon} + ig\Delta x_\mu A_\mu}, \quad \Delta x = x_n - x_{n-1}. \quad \text{(A.4)}$$

Insertion of (A.4) in (A.2) finally yields

$$S = (m - \hat{D}) \int_0^\infty ds e^{-sm^2} (Dz)_{xy} e^{-\frac{1}{4}\int_0^s \dot{z}_\mu^2 d\tau + ig\int_y^x A_\mu dz_\mu + g\int_0^s \sigma_{\mu\nu} F_{\mu\nu} d\tau} \quad \text{(A.5)}$$

where we have used the relation $\hat{D}^2 = D_\mu^2 + g\sigma_{\mu\nu}F_{\mu\nu}$.

Note that in FFSR (A.5) the exponent contains $\gamma$-martices only in the spin term $\sigma_{\mu\nu}F_{\mu\nu}$, and moreover it commutes with $\gamma_5$. Therefore in the chiral limit $(m \to 0)$ $S$ is odd in $\gamma_\mu$ irrespectively of any vacuum averaging of terms containing $A_\mu$ and $F_{\mu\nu}$. Hence in this form one cannot describe the effect of the chiral symmetry breaking and one should look for other representations which will be the topic of other publications.

We start with the case of the free quark and write the Green's function in the energy and the time-dependent representations $S(E)$ and $S(t)$ (in the Minkowskian space-time)

$$S(E) = \frac{1}{m+i\hat{p}} = \frac{\beta}{\hat{H}-E} = \frac{\beta}{m\beta+\boldsymbol{\alpha}\mathbf{p}-E}; \quad S(t) = \int_{-\infty}^{\infty} S(E) e^{-iEt}\frac{dE}{2\pi}. \tag{A.6}$$

Consider now the Foldy-Wouthuyzen (FW) transformation of the free Hamiltonian

$$U^+\hat{H}U = U^+ \begin{pmatrix} m & \boldsymbol{\sigma}\mathbf{p} \\ \boldsymbol{\sigma}\mathbf{p} & -m \end{pmatrix} U \equiv \hat{H}_d = \begin{pmatrix} \sqrt{\mathbf{p}^2+m^2} & 0 \\ 0 & -\sqrt{\mathbf{p}^2+m^2} \end{pmatrix} \tag{A.7}$$

where

$$U = \begin{pmatrix} \cos\hat{\theta} & -\sin\hat{\theta} \\ \sin\hat{\theta} & \cos\hat{\theta} \end{pmatrix}, \quad U^+ = \begin{pmatrix} \cos\hat{\theta} & \sin\hat{\theta} \\ -\sin\hat{\theta} & \cos\hat{\theta} \end{pmatrix}, \tag{A.8}$$

and

$$\sin 2\hat{\theta} = \frac{\boldsymbol{\sigma}\mathbf{p}}{\sqrt{\mathbf{p}^2+m^2}}, \cos 2\hat{\theta} = \frac{m}{\sqrt{\mathbf{p}^2+m^2}}, \quad \hat{\theta} = \frac{1}{2}\arctan(\frac{\boldsymbol{\sigma}\mathbf{p}}{m}). \tag{A.9}$$

Consequently one has

$$S(E) = \beta U \frac{1}{\hat{H}_d - E} U^+, \quad S(t) = \frac{\beta}{2\pi} U \int_{-\infty}^{\infty} \frac{dE(\hat{H}_d+E)}{\mathbf{p}^2+m^2-E^2} U^+ e^{-iEt} \tag{A.10}$$

Integrating in (A.10) one gets finally

$$S(t) = i\beta U \begin{pmatrix} \theta(t) & \\ & -\theta(-t) \end{pmatrix} U^+ e^{-i\sqrt{\mathbf{p}^2+m^2}|t|}. \tag{A.11}$$

Another form can be given to $S(E)$ using the proper-time representation

$$S(E) = \frac{1}{m+i\hat{p}} = i\beta \int_0^{\infty} e^{-i(m\beta+\boldsymbol{\alpha}\mathbf{p}+ip_4)s} ds = i\beta U \int_0^{\infty} e^{-i(\hat{H}_d+ip_4)s} ds U^+. \tag{A.12}$$

The form (A.12) is interesting since it contains the matrix Hamiltonian in the exponent and we shall use it now to take into account external field $A_\mu$. The form (A.7) is especially convenient in the nonrelativistic case when $|\mathbf{p}| \ll m$, and then also $\theta \ll 1$, and the FW transformation is nearly diagonal.

In the opposite case, $|\mathbf{p}| \gg m$, one needs to start from another representation of $\gamma$-matrices, namely the Weyl representation:

$$(E - \mathbf{p}\boldsymbol{\sigma}_W - m\gamma_0^{(W)})\psi = 0; \quad \boldsymbol{\sigma}_W = \begin{pmatrix} \boldsymbol{\sigma}, 0 \\ 0, -\boldsymbol{\sigma} \end{pmatrix}, \gamma_0^{(W)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{A.13}$$

so that the Green's function in the Weyl representation is

$$S_W(E) = (\boldsymbol{\sigma}_W \mathbf{p} + m\gamma_0 - E)^{-1}. \tag{A.14}$$

Doing the FW transformation one has similarly to (A.7)

$$U_W^+ \hat{H}_W U_W = \hat{H}_d^{(W)} = \begin{pmatrix} \sqrt{\mathbf{p}^2 + m^2}\frac{\boldsymbol{\sigma}\mathbf{p}}{|\mathbf{p}|}, & 0 \\ 0, & -\sqrt{\mathbf{p}^2 + m^2}\frac{\boldsymbol{\sigma}\mathbf{p}}{|\mathbf{p}|} \end{pmatrix} \tag{A.15}$$

where

$$U_W = \begin{pmatrix} \cos\theta_W, -\sin\theta_W \\ \sin\theta_W, \cos\theta_W \end{pmatrix} \equiv e^{-it_2\theta_W}, \quad \theta_W = \frac{1}{2}\arctan\frac{m\boldsymbol{\sigma}\mathbf{p}}{\mathbf{p}^2} \tag{A.16}$$

and $t_2 \equiv \sigma_2$ is the Pauli matrix in the helicity indices.

We now use the proper-time representation for $S_W$

$$S_W = \langle x| \int_0^\infty ds e^{is(\boldsymbol{\sigma}_W \boldsymbol{\mathcal{P}} + m\gamma_0 + i\mathcal{P}_4)}|y\rangle \tag{A.17}$$

where $\mathcal{P}_\mu = \frac{1}{i}\partial_\mu - gA_\mu$, and split the interval $(x, y)$ into $N$ steps as in (A.2),$N\varepsilon = s$. One has

$$I_{n,n-1}^{(W)} \equiv \langle x_n|e^{i\varepsilon(\boldsymbol{\sigma}_W \boldsymbol{\mathcal{P}} + m\gamma_0 + i\mathcal{P}_4)}|x_{n-1}\rangle =$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{ip(x_n - x_{n-1})} U_W(\theta_W(n)) e^{i\varepsilon(\hat{H}_D^{(W)}(n) + i\mathcal{P}_4(n))} U_W^+(\theta_W(n)). \tag{A.18}$$

As in (A.3) one can write $\mathcal{P}_\mu = p_\mu - gA_\mu$, and integrate over $d^4p$, representing the square root terms in $\hat{H}_d^{(W)}$ through the einbein function $\mu(x_4)$,

$$e^{i\varepsilon\sqrt{\mathbf{p} + m^2}} \sim \int d\mu_n e^{i\left(\frac{\mathbf{p}^2 + m^2}{2\mu_n} + \frac{\mu_n}{2}\right)\varepsilon} \tag{A.19}$$

one can also use identity;

$$e^{ia\boldsymbol{\sigma}\mathbf{n}} = e^{ia}\frac{(1 + \boldsymbol{\sigma}\mathbf{n})}{2} + e^{-ia}\frac{(1 - \boldsymbol{\sigma}\mathbf{n})}{2}. \tag{A.20}$$

Now the integration over $d^4p$ yields

$$I^{(w)}_{n,n-1} = \int d\mu_n \left(\frac{\mu_n}{2\pi\varepsilon}\right)^2 U_W(\theta_W) e^{igA_\mu \Delta x_\mu} G(n) U^+_W(\theta_W) \tag{A.21}$$

where we have defined the diagonal matrix $G(n)$ with elements

$$G_{++}(n) = e^{-\Delta x_4[\frac{\mu}{2}(\dot{\mathbf{x}}^2+1)+\frac{m^2}{2\mu}]}\theta(\Delta x_4)\frac{1+\boldsymbol{\sigma}\mathbf{n}}{2} + e^{\Delta x_4[\frac{\mu}{2}(\dot{\mathbf{x}}^2+1)+\frac{m^2}{2\mu}]}\theta(-\Delta x_4)\frac{1-\sigma\mathbf{n}}{2}$$

$$G_{--}(n) = e^{-\Delta x_4[\frac{\mu}{2}(\dot{x}^2+1)+\frac{m^2}{2\mu}]}\theta(\Delta x_4)\frac{1-\boldsymbol{\sigma}\mathbf{n}}{2} + e^{\Delta x_4[\frac{\mu}{2}(\dot{\mathbf{x}}^2+1)+\frac{m^2}{2\mu}]}\frac{1+\boldsymbol{\sigma}\mathbf{n}}{2}\theta(-\Delta x_4) \tag{A.22}$$

and $\dot{\mathbf{x}} = \frac{\Delta\mathbf{x}}{\Delta x_4},\ \Delta x_\mu = (x_n - x_{n-1})_\mu$, while $\mathbf{p}$ residing in $\theta_W$ is $\mathbf{p} = \mu\dot{\mathbf{x}}$.

## Appendix 2

The function $\varphi(t), t \equiv m/\delta$, defined in Eq.(29) can be written as (note the difference in definition here and in [9])

$$\varphi(t) = t \int_0^\infty z^2 dz K_1(tz) e^{-z} \tag{A2.1}$$

where $K_1$ is the McDonald function, $K_1(x)(x \to 0) \approx \frac{1}{x}$, so that for $t = 0$ one obtains

$$\varphi(0) = 1. \tag{A2.2}$$

For $t > 0$ the integration in (A2.1) yields two different forms; e.g. for $t < 1$,

$$\varphi(t) = -\frac{3t^2}{(1-t^2)^{5/2}} \ln \frac{1+\sqrt{1-t^2}}{t} + \frac{1+2t^2}{(1-t^2)^2} \tag{A2.3}$$

while for $t > 1$ one has instead,

$$\varphi(t) = -\frac{3t^2}{(t^2-1)^{5/2}} \arctan(\sqrt{t^2-1}) + \frac{1+2t^2}{(1-t^2)^2}. \tag{A2.4}$$

For large $t$ one has the following limiting behaviour,

$$\varphi(t) = \frac{2}{t^2} - \frac{3\pi}{2t^3} + O(\frac{1}{t^4}). \tag{A2.5}$$

For small $t$ one obtains expanding the r.h.s. of (A2.3)

$$\varphi(t) = 1 + t^2(4 - 3\ln\frac{2}{t}) + t^4(\frac{7}{4} - \frac{15}{2}\ln\frac{2}{t}) + O(t^6). \tag{A2.6}$$

Some numerical values are useful in applications.

$$\varphi(0.175) \cong 0.88, \quad \varphi(1.7) \cong 0.234, \varphi(5) \cong 0.052$$

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